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Nonperturbative aspects of screening phenomena in gauge theories: Nucl. Phys. B459, 559 (1996)
Lifetime of quasiparticles in hot QED plasmas: Phys. Rev. Lett. 76, 3080 (1996)
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J. P. Blaizot, E. Iancu & A. Rebhan: Resumming Hard Thermal Loops

The Entropy of the QCD plasma: Phys. Rev. Lett. 83, 2906 (1999)
Selfconsistent HTL thermodynamics for the quark gluon plasma: Phys. Lett. B470, 181 (1999)
Approximate resummations for QGP: Entropy and density: Phys. Rev. D63, 065003 (2001)
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Thermodynamics of the high temperature QGP, hep-ph/0303185

> "...this approach reproduces accurately the entropy obtained from lattice gauge calculations at temperatures above 2.5 T_c. This calculation thus provides also support to the physical picture of the quark-gluon plasma as a gas of weakly interacting quasiparticles."

HTL's and the entropy of SUSY Yang-Mills theories: JHEP 0706, 035 (2007) (With U.Kraemmer)

When is nuclear matter quarkyonic?

McLerran & RDP, 0706.2191.

Hidaka, McLerran, & RDP 0803.0279

Hidaka, Kojo, McLerran, & RDP ’09...

Blaizot, Nowak, McLerran & RDP ’09...

Always at large N_c, *maybe* for $N_c = 3$

Or: the unbearable lightness of nuclear matter...

Towards the phase diagram at $N_c = \infty$

't Hooft '74: let $N_c \to \infty$, with $\lambda = g^2 N_c$ fixed.
E.g.: gluon polarization tensor at zero momentum.

$$\Pi^{\mu\mu}(0) = g^2 \left(\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} \right) = \lambda \, \frac{T^2}{3} \ , \ N_c = \infty$$

For $\mu \sim N_c^0 \sim 1$, at $N_c = \infty$ the gluons are *blind* to quarks.

When $\mu \sim 1$, since gluons don't feel quarks, the
deconfining transition temperature is independent of μ! $T_d(\mu) = T_d(0)$

Chemical potential only matters when larger than mass:
$\mu_{Baryon} > M_{Baryon}$. Define $m_{quark} = M_{Baryon}/N_c$; so $\mu > m_{quark}$.

"Box" for $T < T_c$; $\mu < m_{quark}$: confined phase baryon free, since their mass $\sim N_c$
Thermal excitation $\sim \exp(-m_B/T) \sim \exp(-N_c) = 0$ at large N_c.
So hadronic phase in "box" = mesons & glueballs only, *no* baryons.

Phase diagram at $N_c = \infty$, I

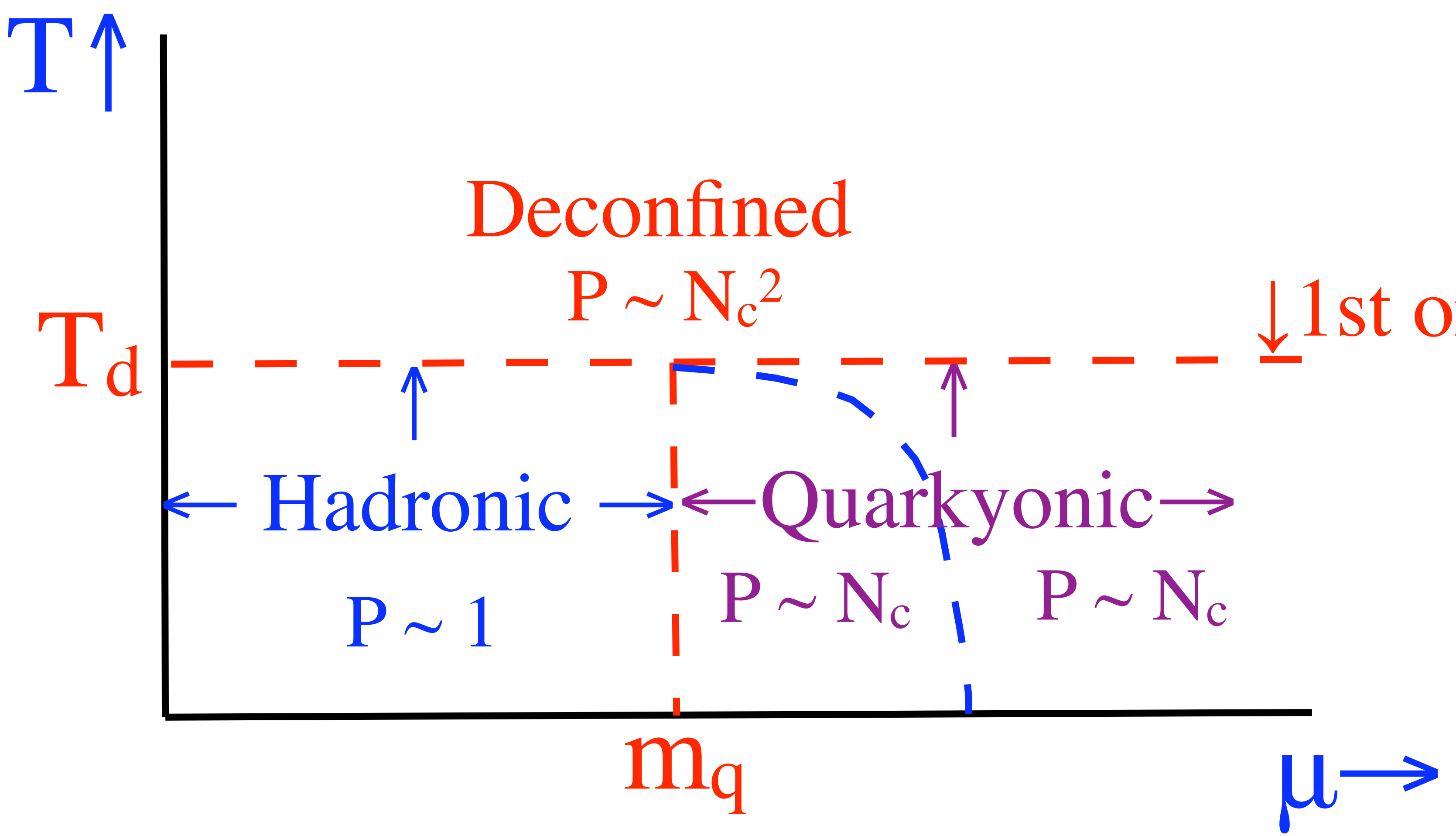
At *least* three phases. At large N_c, can use pressure, P, as order parameter.

Hadronic (confined): $P \sim 1$. Deconfined, $P \sim N_c^2$. Thorn '81; RDP '84...

$P \sim N_c$: quarks or baryonic = "quark-yonic". Chiral symmetry restoration?

L. McLerran & RDP, 0706.2191

N.B.: mass threshold at m_q neglects (possible) nuclear binding, Son.

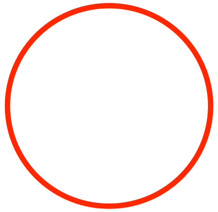


Nuclear matter at large N_c

$\mu_{Baryon} = \sqrt{k_F^2 + M^2}$, k_F = Fermi momentum of baryons.

Pressure of ideal baryons density times energy of non-relativistic baryons:

$$P_{\text{ideal baryons}} \sim n(k_F)\,\frac{k_F^2}{M} \sim \frac{1}{N_c}\,\frac{k_F^5}{\Lambda_{QCD}}$$

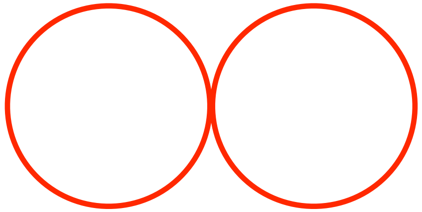


This is small, ~ $1/N_c$. The pressure of the I = J tower of resonances is as small:

$$\delta P_{\text{resonances}} \sim \frac{1}{M}\,\frac{k_F^8}{\Lambda_{QCD}^3} \sim \frac{1}{N_c}\,\frac{k_F^8}{\Lambda_{QCD}^4}$$

Two body interactions are huge, ~ N_c in pressure.

$$\delta P_{\text{two body int.'s}} \sim N_c\,\frac{n(k_F)^2}{\Lambda_{QCD}^2} \sim N_c\,\frac{k_F^6}{\Lambda_{QCD}^2}$$



At large N_c , nuclear matter is dominated by potential, not kinetic terms!

Two body, three body... interactions *all* contribute ~ N_c .

N.B.: these are all *contact* interactions.

Window of nuclear matter

Balancing $P_{\text{ideal baryons}} \sim P_{\text{two body int.'s}}$, interactions important very quickly,

$$k_F \sim \frac{1}{N_c^2} \Lambda_{QCD}$$

For such momenta, only two body interactions contribute.

By the time $k_F \sim 1$, *all* interactions terms contribute $\sim N_c$ to the pressure.

But this is *very* close to the mass threshold,

$$\mu - m_q = \frac{\mu_B - M}{N_c} = \frac{k_F^2}{2MN_c} \sim \frac{1}{N_c^2} k_F^2$$

Hence "ordinary" nuclear matter is only in a *very* narrow window.

One quickly goes to a phase with pressure $P \sim N_c$.

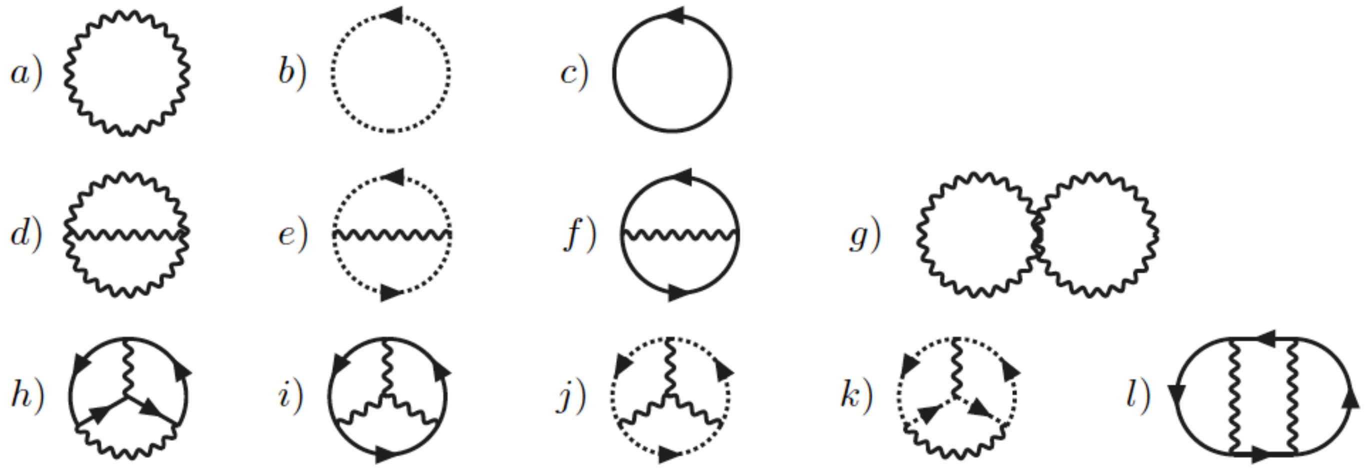
So are they baryons, or quarks?

Perturbative pressure

At high density, $\mu >> \Lambda_{QCD}$, compute $P(\mu)$ in QCD perturbation theory.

To $\sim g^4$, (Freedman & McLerran)^4 '77

Ipp, Kajantie, Rebhan, & Vuorinen, hep-ph/0604060



At $\mu \neq 0$, only diagrams with at least one quark loop contribute. Still...

$$P_{\text{pert.}}(\mu) \sim N_c N_f \; \mu^4 \; F_0(g^2(\mu/\Lambda_{QCD}), N_f)$$

For $\mu >> \Lambda_{QCD}$, but $\mu \sim N_c^0 \sim 1$, calculation reliable.

Compute $P(\mu)$ to $\sim g^6$? No "magnetic mass" at $\mu \neq 0$, well defined $\forall \; (g^2)^n$.

"Quarkyonic" phase at large N_c

As gluons blind to quarks at large N_c, for $\mu \sim N_c^0 \sim 1$, *confined* phase for $T < T_d$

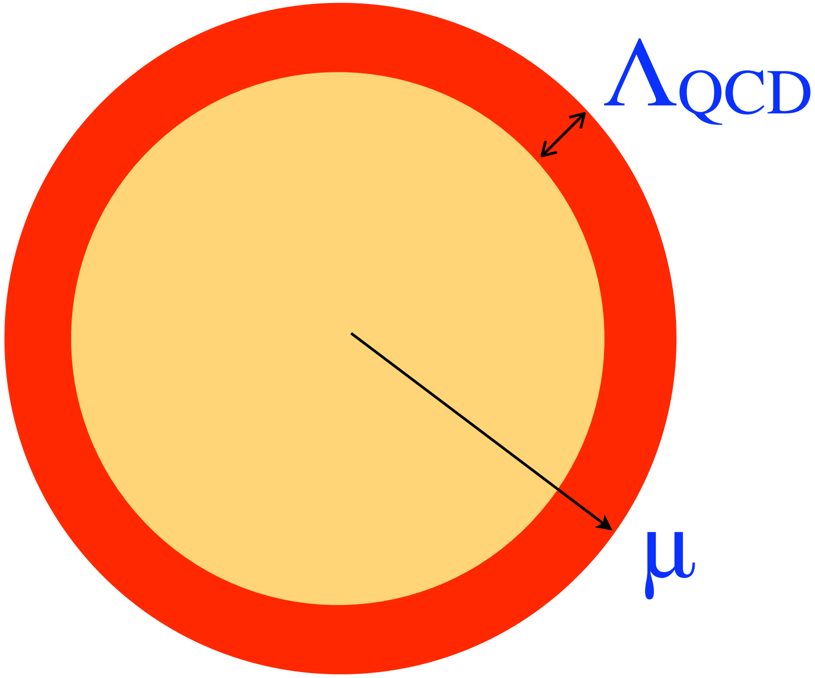
This includes $\mu >> \Lambda_{QCD}$! Central puzzle. We suggest:

To the right: Fermi sea =>

Deep in the Fermi sea, $k << \mu$,
looks like quarks.

But: within ~ Λ_{QCD} of the Fermi surface,
confinement => *baryons*

We term combination "*quark-yonic*"



OK for $\mu >> \Lambda_{QCD}$. When $\mu \sim \Lambda_{QCD}$, baryonic "skin" entire Fermi sea.

But what about chiral symmetry breaking?

Skyrmions and $N_c = \infty$ baryons

Witten '83; Adkins, Nappi, Witten '83: Skyrme model for baryons

$$\mathcal{L} = f_\pi^2 \, \mathrm{tr}|V_\mu|^2 + \kappa \, \mathrm{tr}[V_\mu, V_\nu]^2 \; , \; V_\mu = U^\dagger \partial_\mu U \; , \; U = e^{i\pi/f_\pi}$$

Baryon soliton of pion Lagrangian: $f_\pi \sim N_c^{1/2}$, $\kappa \sim N_c$, mass $\sim f_\pi^2 \sim \kappa \sim N_c$.

Above Lagrangian simplest form: presumably (?) *infinite* series in V_μ.

Single baryon: at $r = \infty$, $\pi^a = 0$, $U = 1$. At $r = 0$, $\pi^a = \pi\, r^a/r$.
Baryon number topological: Wess & Zumino '71; Witten '83.

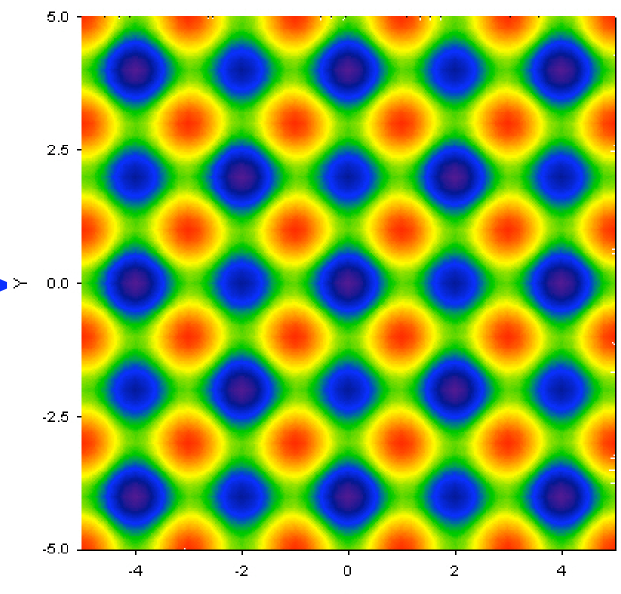
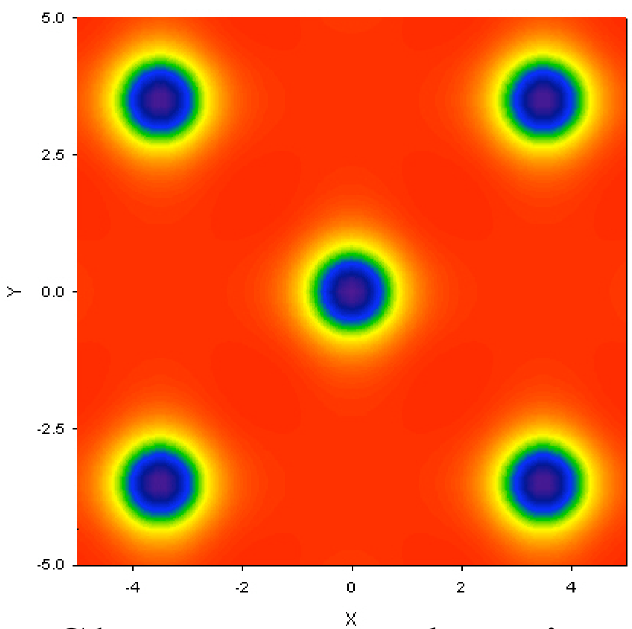
Huge degeneracy of baryons: multiplets of isospin and spin, $I = J$: $1/2 \ldots N_c/2$.
Obvious as collective coordinates of soliton, coupling spin & isospin

Dashen & Manohar '93, Dashen, Jenkins, & Manohar '94:
Baryon-meson coupling $\sim N_c^{1/2}$,
Cancellations from extended $SU(2\, N_f)$ symmetry.

Skyrmion crystals

Kutschera, Pethick & Ravenhall (KPR) '84; Klebanov '85 + ...
Lee, Park, Min, Rho & Vento, hep-ph/0302019; Park, Lee, & Vento, 0811.3731:

At large N_c, baryons are heavy, so form a crystal.
Form Skyrmion crystal by taking periodic boundary conditions in a box.
Lee+... '03 : box of size L, units of length $1/(\sqrt{\kappa}\, f_\pi)$, plot baryon number density:

L=2.0→   ←L=3.5

At low density, chiral symmetry broken by Skyrme crystal, as in vacuum.

But chiral symmetry *restored* at nonzero L (density): $\langle U \rangle = 0$ in *each* cell.

Skyrmion crystals as quarkyonic matter

Why chiral symmetry restoration in a Skyrmion crystal?

Goldhaber & Manton '87: due to "half" Skyrmion symmetry in each cell.

Easiest to understand with "spherical" crystal: sphere instead of cube...

KPR '84, Ruback & Manton '86, Manton '87. Consider the "trivial" map:

$$U(r) = \exp(i\, f(r)\, \hat{r} \cdot \tau) \ ; \ f(r) = \pi \left(1 - \frac{r}{R}\right)$$

Solution has unit baryon number per unit sphere, and so is a crystal.

Solution is minimal when R < √2 (* 1/(√κ f_π).

Forkel, Jackson, Rho, & Weiss '89 =>

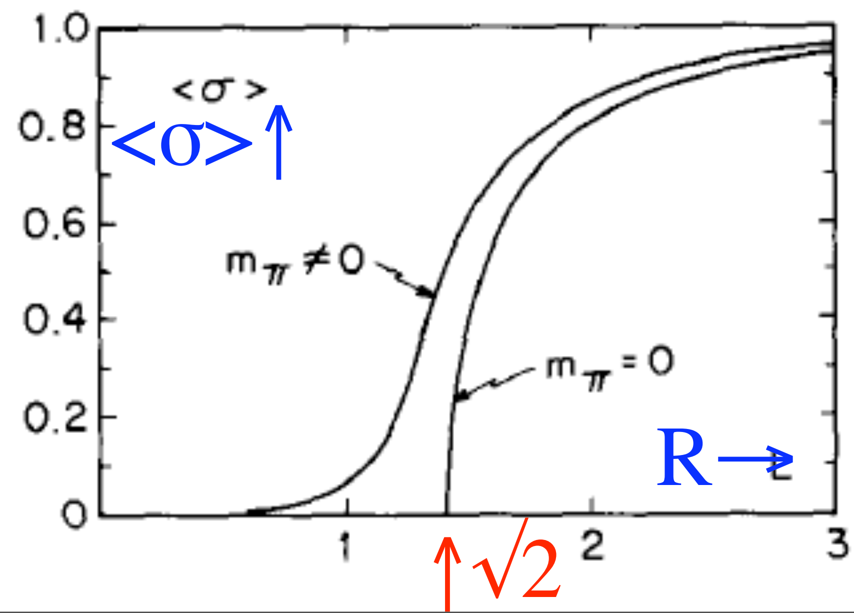
looks like standard chiral transition!

Excitations *are* chirally symmetric.

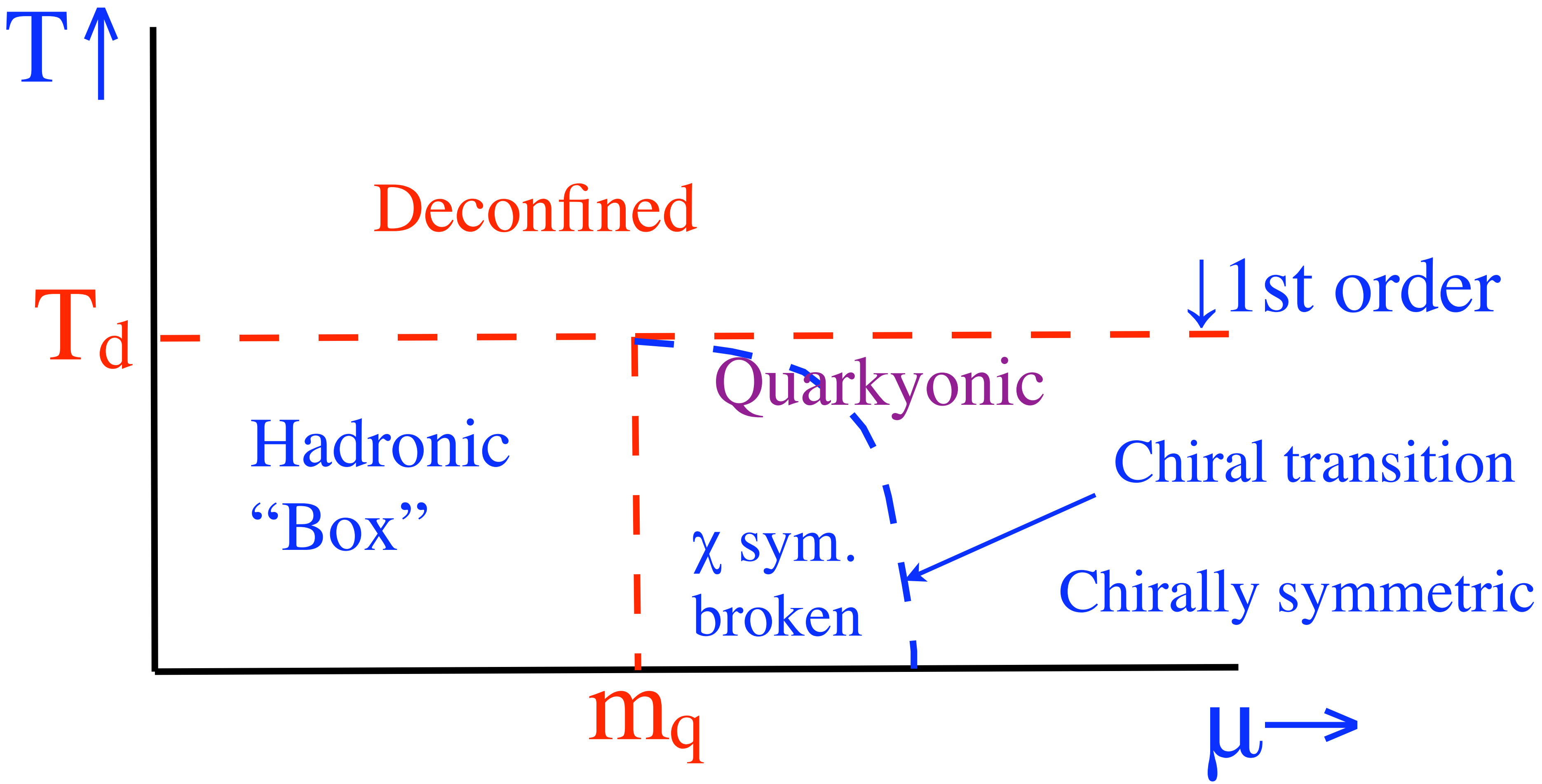
But Skyrmions are *not* deconfined.

Example of quarkyonic matter,

chirally broken and chirally symmetric.



Phase diagram at $N_c = \infty$, II



We suggest: quarkyonic phase includes chiral trans. Order by usual arguments.

Mocsy, Sannino & Tuominen hep-th/0308135:

splitting of transitions in effective models

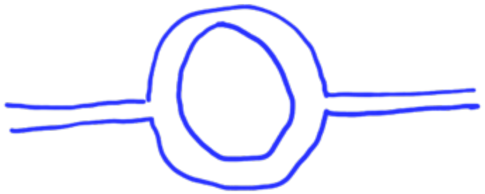
But: quarkyonic phase *confined*. Chirally symmetric baryons?

Baryons at Large N_f

Veneziano '78: take *both* N_c and N_f large. Mesons M^{ij} : $i,j = 1 \ldots N_f$.

Thus mesons interact weakly, but there are *many* mesons.

Thus in the hadronic phase, mesons interact *strongly*:



$$\Pi \sim N_f \, g_{3\pi}^2 \sim N_f / N_c$$

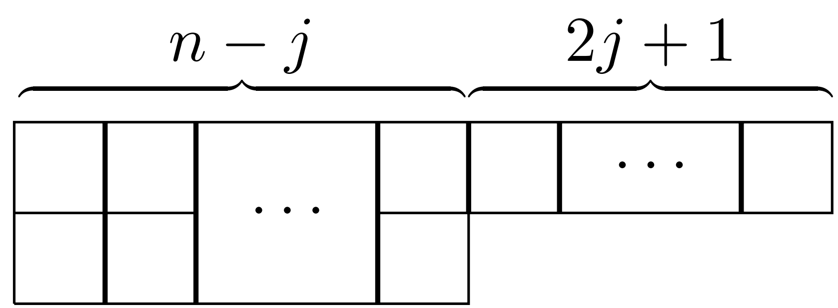
Pressure large in *both* phases:

$\sim N_f^2$ in hadronic phase, $\sim N_c^2$, $N_c N_f$ in "deconfined" phase.

Polyakov loop also nonzero in both phases.

Baryons: lowest state with spin j

has Young tableaux ($N_c = 2n + 1$) =>



$$d_j = \frac{(2j+2)\,(N_f+n+j)!\,(N_f+n-j-2)!}{(N_f-1)!\,(N_f-2)!\,(n+j+2)!\,(n-j)!}$$

Baryons at Large N_f: order parameters

Y. Hidaka, L. McLerran & RDP, 0803.0279: Use Sterling's formula,

$$d_j \sim \mathrm{e}^{+N_c\, f(N_f/n)} \;,\;\; f(x) = (1+x)\log(1+x) - x\log(x)$$

Degeneracy of baryons increases *exponentially*.

Argument is heuristic: baryons are strongly interacting.
Still, difficult to see how interactions can overwhelm exponentially growing spectrum, even for the lowest state.

Use *baryons* as order parameter. At T=0, fluctuations in baryon number,
$\langle B^2 \rangle \neq 0$ when $N_c\, f(N_c/n) = m_B/T$, or

$$T_{qk} = f(N_f/n)\,\frac{m_B}{N_c}$$

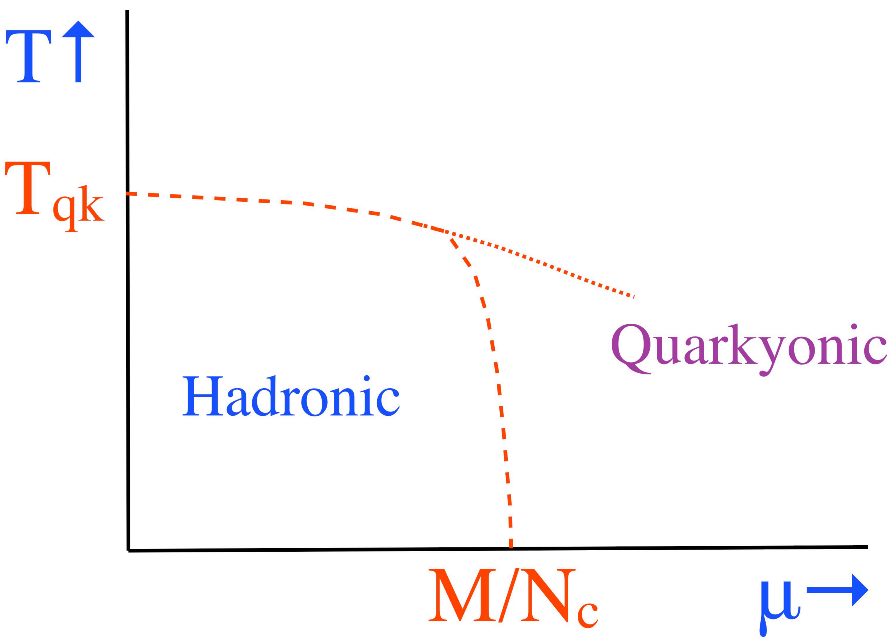
At $\mu \neq 0$, baryon number itself:
$\langle B \rangle \neq 0$ when $N_c\, f(N_c/n) = (m_B - N_c\,\mu)/T$:

$$T_{qk} = f(N_f/n)\,\left(\frac{m_B}{N_c} - \mu\right)$$

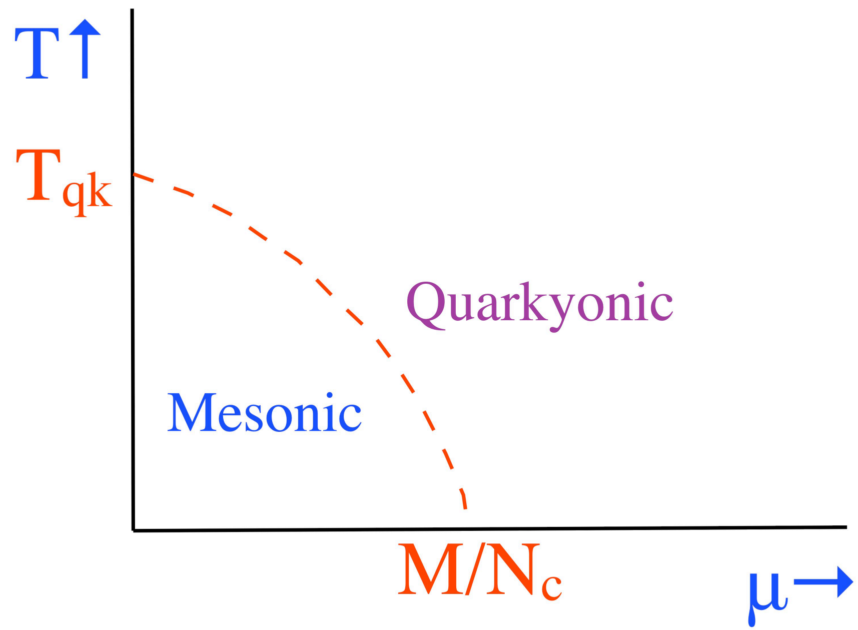
Possible phase diagrams as N_f increases

The "rectangle" for small N_f becomes smoothed.
Eventually, maybe the quarkyonic line merges with that for baryon condensation.
All *VERY* qualitative. Clearly many possible phase diagrams!
With SUSY: condensation of Higgs fields as well.

Small N_f

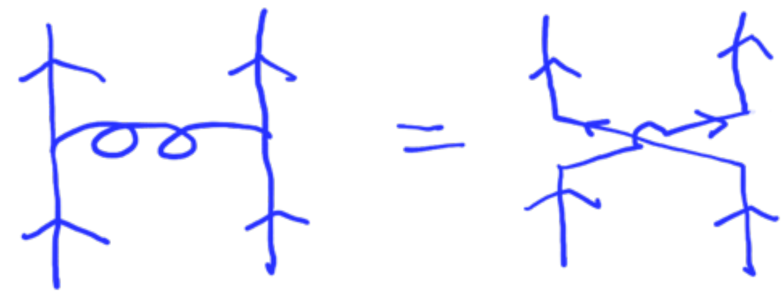


Large N_f



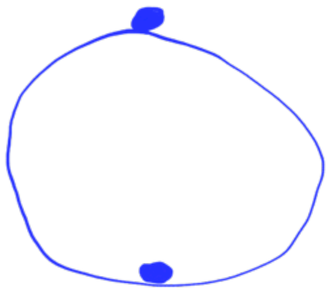
Chiral Density Waves (perturbative)

Excitations near the Fermi surface?



At large N_c, color superconductivity suppressed,
$\sim 1/N_c$: pairing into two-index state:

Also possible to have "chiral density waves", pairing of quark and anti-quark:
Deryagin, Grigoriev, & Rubakov '92. Shuster & Son, hep-ph/9905448.
Rapp, Shuryak, and Zahed, hep-ph/0008207.



Order parameter $\langle \overline{\psi}(-\vec{p}_f)\, \psi(+\vec{p}_f)\rangle$
Sum over color, so *not* suppressed by $1/N_c$.

Pair quark at $+\,p_f$ with anti-quark at $-\,p_f$: for a *fixed* direction.
Breaks chiral symmetry, with state varying $\sim \exp(-\,2\,p_f\,z)$.

Wins over superconductivity in low dimensions. Loses in higher.
Shuster & Son '99: in perturbative regime, CDW only wins for $N_c > 1000\,N_f$

Quarkyonic chiral density waves

Consider meson wave function, with kernel:
Confining potential in 3+1 dimensions like
Coulomb potential in 1+1 dim.s:

$$\int dk_0\, dk_z \int d^2k_\perp \,\frac{1}{(k_0^2+k_z^2+k_\perp^2)^2} \sim \int dk_0\, dk_z\, \frac{1}{k_0^2+k_z^2}$$

In 1+1 dim.'s, behavior of massless quarks near Fermi surface maps ~ $\mu = 0$!

Mesons in vacuum naturally map into CDW mesons.

Witten '84: in 1+1 dim.'s, use non-Abelian bosonization for QCD.

a, b= 1...N_c. i,j = 1... N_f.

$$J_+^{ij} = \overline{\psi}^{a,i}\psi^{a,j} \sim g^{-1}\partial_+ g\;;\; J_+^{ab} = \overline{\psi}^{a,i}\psi^{b,i} \sim h^{-1}\partial_+ h\;.$$

Steinhardt '80. Affleck '86. Frishman & Sonnenschein, hep-th/920717...
Armoni, Frishman, Sonnenschein & Trittman, hep-th/9805155; AFS, hep-th/0011043..
Bringoltz 0901.4035; Galvez, Hietanan, & Narayanan, 0812.3449.

Bosonized quarkyonic matter

After non-Abelian bosonization, action factorizes into sum of g, in SU(N_f), and h, in SU(N_c). Action for g is

$$8\pi\, S_{WZW} = \int d^2z\, \mathrm{tr}\, B_i^2 + 2/3 \int d^3y\, \epsilon^{ijk}\, \mathrm{tr}\, B_i B_j B_k \ , \ B_i = g^{-1}\partial_i g \ .$$

Action for h, is a SU(N_c) gauged WZW model. But: g and h *decouple*!
Spectrum of h complicated, involves massive modes, like usual 't Hooft model.

Spectrum of g is that of usual WZW model, with *massless* modes.

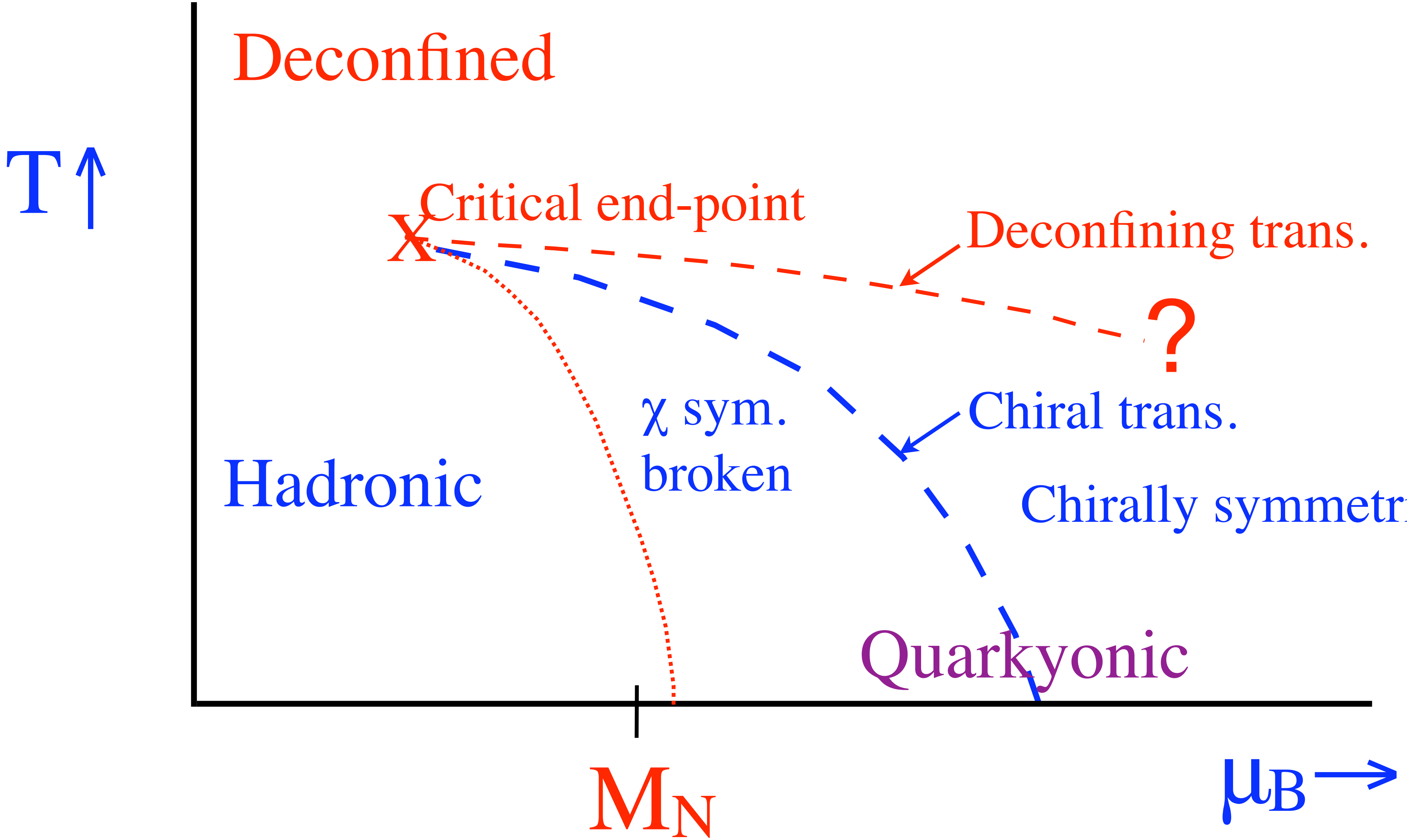
Hence in 1+1 dim.'s, CDW are natural, but with *massless* excitations thereof.

In 3+1 dim.'s: have highly anisotropic state, *somen*-state:
Y. Hidaka, T. Kojo, L. McLerran, & RDP '09...

Chiral condensate ~ Λ_{QCD}^2/μ^2. Length of somen-state large, ~ exp(N_c).
Quantum fluctuations tend to scramble the somen.

Guess for phase diagram in QCD

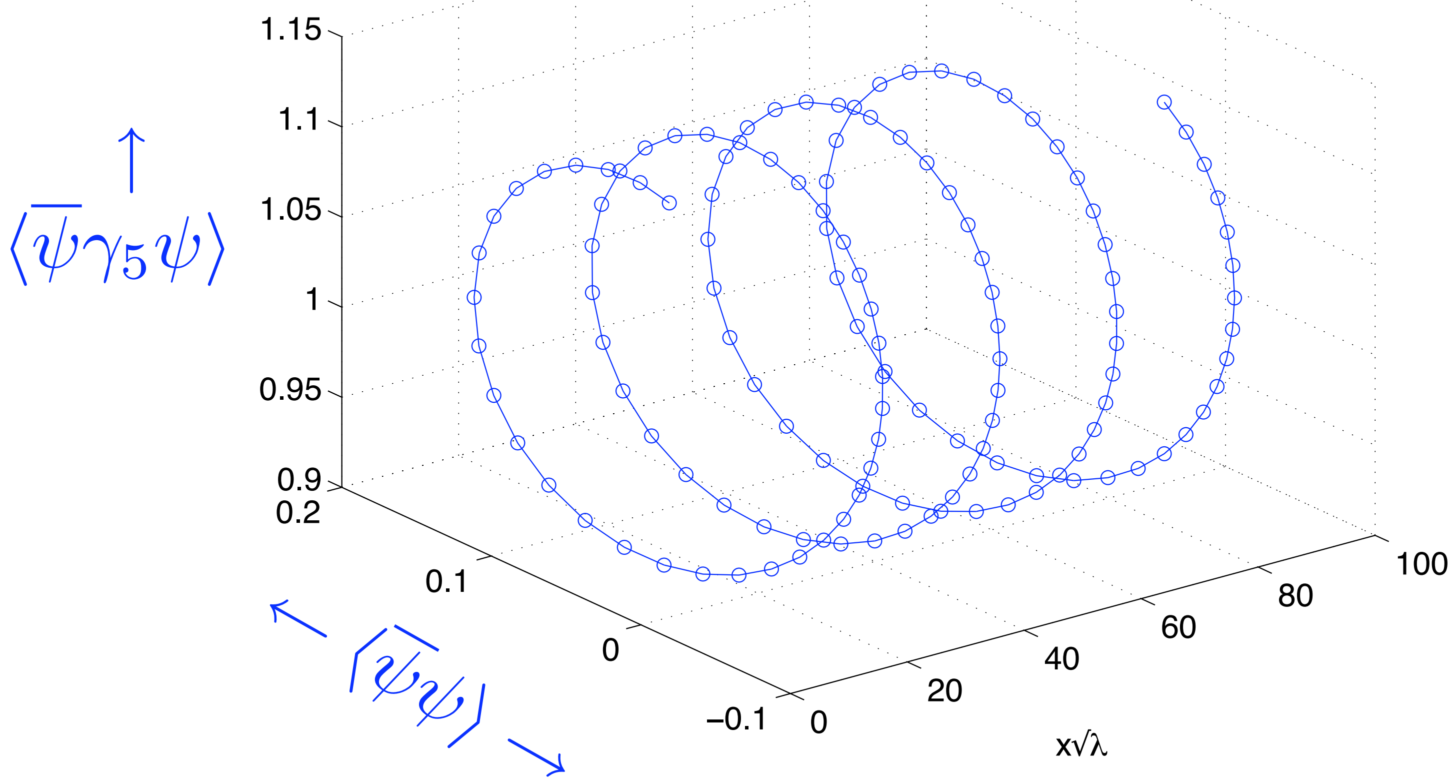
Pure guesswork: deconfining & chiral transitions split apart at critical end-point?
Line for deconfining transition first order to the right of the critical end-point?
Critical end-point for deconfinement, or continues down to T=0?



Solution to dense QCD in 1+1 dimensions

Bringoltz, 0901.4035: 't Hooft model, with massive quarks.
Works in Coulomb gauge, in *canonical* ensemble: fixed baryon number.
Solves numerically equations of motion under constaint of nonzero baryon #
Finds chiral density wave.

N.B.: for massive quarks, should have massless excitations, but with energy $\sim 1/N_c$.



Lattice: nucleon-nucleon potentials

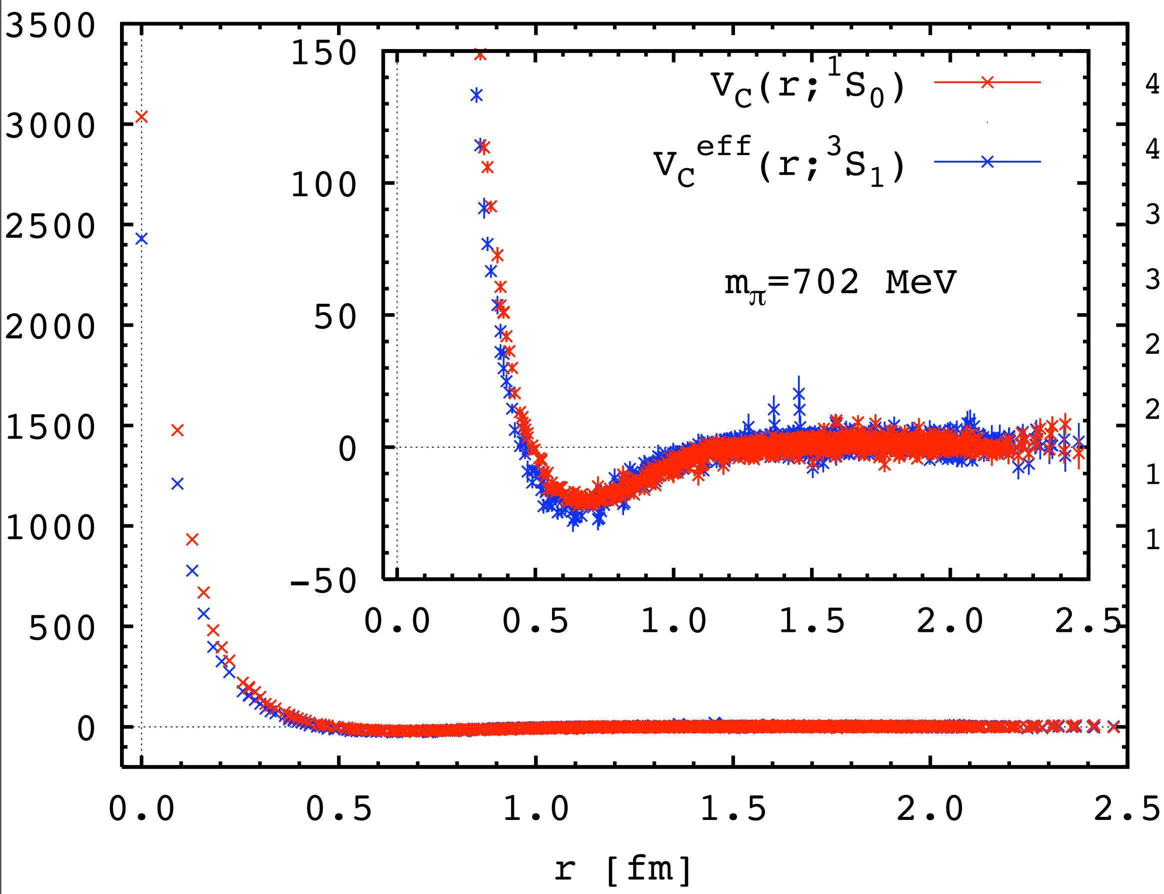
Ishii, Aoki & Hatsuda, PACS-CS, 0903.5497

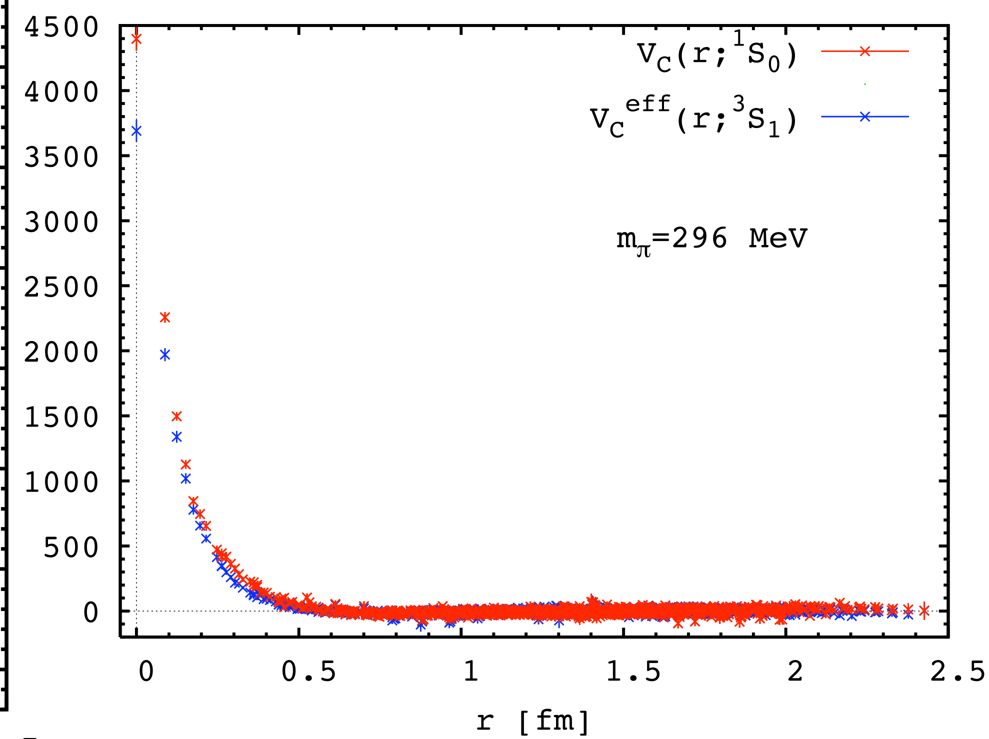
Nucleon-nucleon potentials from quenched and 2+1 flavors.

Pions heavy: 700 MeV (left) and 300 MeV (right)

Essentially *zero* potential plus strong hard core repulsion

(hard core stronger with dynamical quarks)





Lattice: Lambda-proton potentials

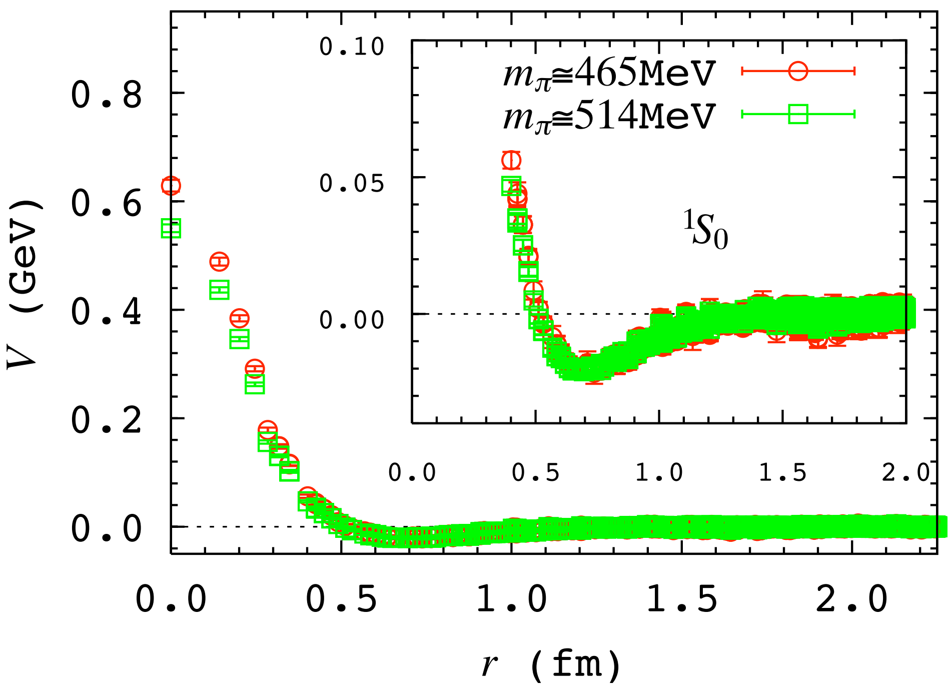
Nemura, Ishii, Aoki & Hatsuda, PACS-CS, 0902.1251

Nucleon-nucleon potentials from quenched and 2+1 flavors.

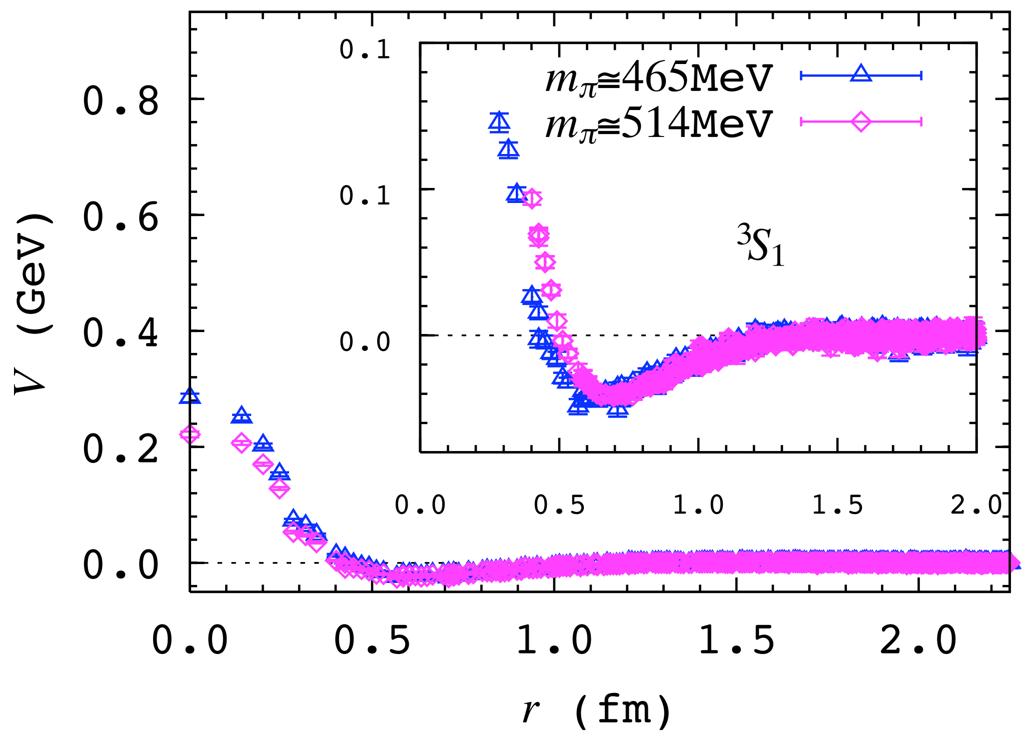
Pion ~ 300 MeV, Kaon ~ 600 MeV.

Again, essentially *zero* potential plus strong hard core repulsion.

$^1S_0$



$^3S_1$



Purely pionic nuclear matter

J.-P. Blaizot, L. McLerran, Maciek Nowak, & RDP '09....

At infinite N_c, integrate out *all* degrees of freedom *except* pions.

Power series in $V_\mu = U^\dagger \partial_\mu U \; , \; U = e^{i\pi/f_\pi}$

Not just the usual Skyrme model. Infinite set of couplings, including terms with more than two time derivatives. At fourth order, add: $\left(\mathrm{tr}\; V_\mu^2\right)^2$ which is part of the usual Gasser-Leutwyler expansion.

At sixth order, have $(J_\alpha)^2 \quad ; \quad J_\alpha = \mathrm{tr}\,(V_\beta V_\gamma V_\delta)$
from integrating out the ω meson. This has only two times derivatives.

Other terms at sixth order: $\left(\mathrm{tr}\; V_\mu^2\right)^3 \quad , \quad \left(\mathrm{tr}\; V_\mu^2\right) \; \mathrm{tr}[V_\mu, V_\nu]^2 \quad \ldots$

Do *not* worry about higher time derivatives.

Use *only* as effective theory valid for momenta less than lightest particle above the pion & kaon: sigma, omega, rho...

Baryons at infinite N_c

Let the effective, purely pionic Lagrangian be $\mathcal{L}_\pi(V_\mu)$

Take each and every coupling to be proportional to N_c. Then we can compute a "purely" pionic baryon as the Skyrmion in infinite space, and a baryonic crystal in a box.

Consistency? Not clear. But have an infinite number of couplings, and no way to obtain their values. Further, all couplings contribute equally to the nucleon mass, properties of the crystal, etc.

So? At infinite N_c, $f_\pi \sim \sqrt{N_c}$ is *large*. (Why $4\pi f_\pi$ expansion parameter of χPT?)

At low momentum, baryons interact as $\overline{\psi}\left(i\partial\!\!\!/ + M_B\, \mathrm{e}^{i\tau\cdot\pi\gamma_5/f_\pi}\right)\psi$

Weinberg '67: by field redefinition, this becomes

$$\overline{\psi}\left(iW\,\partial\!\!\!/\,W^{-1} + M_B\right)\psi \;\;,\;\; W = \mathrm{e}^{i\tau\cdot\pi\gamma_5/2f_\pi}$$

Soft pions and baryons at infinite N_c

Expanding in powers of pions, $$\mathcal{L}_{\rm int} \sim \frac{1}{f_\pi}\,\overline{\psi}\,\gamma_5 \tau \cdot \partial\!\!\!/ \pi\,\psi + \ldots$$

Sure. So what? Valid when momenta are << f_π: but this is big!

Thus at infinite N_c, *if* there is a purely pionic effective Lagrangian, the baryons are *free*. Breaks down at distances $1/f_\pi$: short range repulsion?

Crystal: energy/baryon is *zero*, except when the density is > $1/f_\pi^3$.

Depends *crucially* upon existence of purely pionic effective Lagrangian; else exchange of other hadrons destroys computation.

Hence: without chiral symmetry, expect strong interactions ~ N_c.
Seen in the 't Hooft model in 1+1 dimensions: energy/baryon ~ N_c.

Related: Goldberger-Trieman at large N_c?
Is g_A of order one or order N_c?

$$g_A = g_{\pi NN}/(2 f_\pi M_B)$$

Asymptotically large μ, grows with N_c

For $\mu \sim (N_c)^p$, $p > 0$, gluons feel the effect of quarks. Perturbatively,

$$P_{\text{pert.}}(\mu, T) \sim N_c N_f \, \mu^4 \, F_0 \; , \; N_c N_f \, \mu^2 \, T^2 \, F_1 \; , \; N_c^2 \, T^4 \, F_2 \; .$$

First two terms from quarks & gluons, last only from gluons. Two regimes:

$\mu \sim N_c^{1/4} \Lambda_{QCD}$: $N_c \, \mu^4 \, F_0 \sim N_c^2 \, F_2 \sim N_c^2 >> N_c \, \mu^2 \, F_1 \sim N_c^{3/2}$.

Gluons & quarks contribute equally to pressure; quark cont. T-independent.

$\mu \sim N_c^{1/2} \; \Lambda_{QCD}$: New regime: $m^2_{Debye} \sim g^2 \, \mu^2 \sim 1$, so gluons feel quarks.

$N_c \, \mu^4 \, F_0 \sim N_c^3 >> N_c \, \mu^2 \, F_1$, $N_c^2 \, F_2 \sim N_c^2$.

Quarks dominate pressure, T-independent.

Eventually, first order deconfining transition can either:

end in a critical point, or bend over to T = 0: ?